Submodular Dictionary Learning for Sparse Coding

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Goals

- Motivations

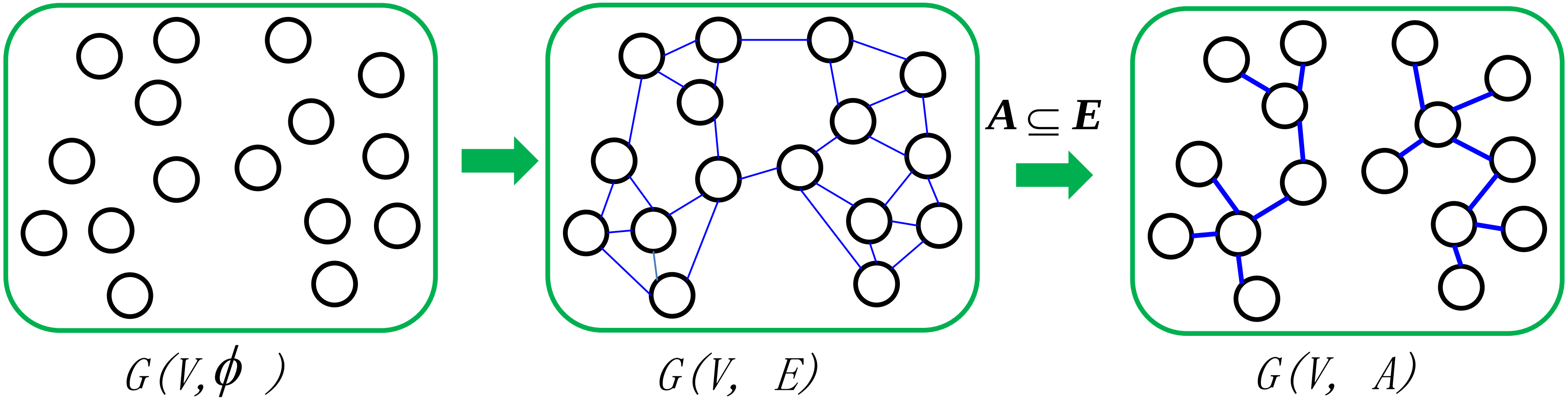
  Most of recent dictionary learning techniques are iterative batch procedures, it is relatively slow close to the minimum.

- Goals

  - Learn a discriminative and representational dictionary for sparse representation efficiently using a greedy algorithm for a submodular objective set function.

Approaches

- Approaches
  - A dataset is mapped into an undirected k-nearest neighbor graph G=(V, E). The dictionary learning is modeled as a graph topology selection problem. A subset of edges A is selected from initial edge set E such that the resulting graph G=(V, A), contains exactly K connected components or clusters.



Approaches

- Approaches
  - A monotonic and submodular objective function for dictionary learning consists of two terms: the entropy rate of a random walk on a graph and a discriminative term
  - The objective function is optimized by a highly efficient greedy algorithm
  - This simple greedy algorithm gives a near-optimal solution with a (1/2)-approximation bound [5].

Related Work

- Sparse Coding has been successfully applied to a variety of problems such as face recognition [1]. The SRC algorithm [1] employs the entire set of training samples to form a dictionary.
- K-SVD [2]: Efficiently learn an over-complete dictionary with a small size. It focuses on representational power, but it does not consider discrimination.
- Discriminative dictionary learning approaches:
  - Constructing a separate dictionary for each class.
  - Adding discriminative terms into the objective function of dictionary learning [3].
- The diminishing return property of a submodular function has been employed in applications such as sensor placement, clustering and superpixel segmentation [4].

Preliminaries

- Submodular Set Function

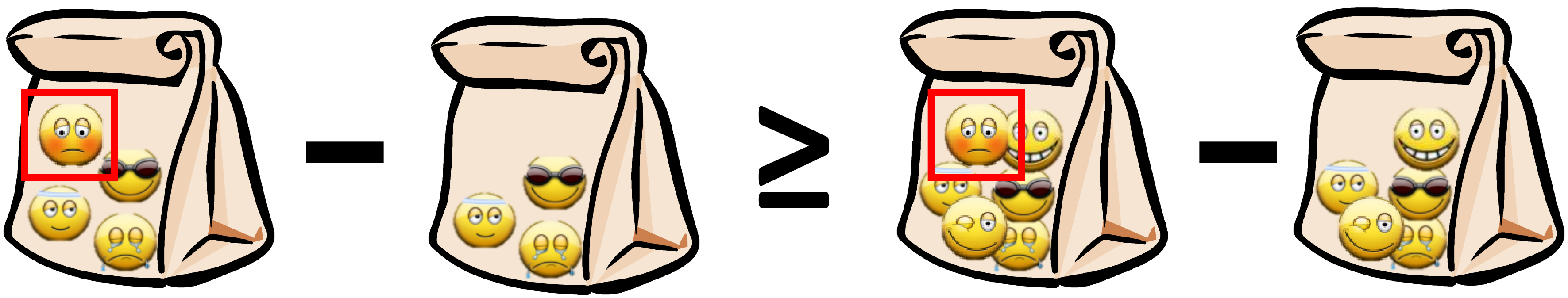
A set function $F : 2^E \rightarrow R$ is submodular if

$$F(A_1 \cup \{a\}) - F(A_1) \geq F(A_2 \cup \ a) - F(A_2)$$

for all $A_1 \subseteq A_2 \subseteq E$ and $a \in E \backslash A_2$

diminishing return property

$F(\ A_1\ U\ \{a\}\ )\ -\ F(\ A_1\ )\ \geq\ F(\ A_2\ U\ \{a\}\ )\ -\ F(\ A_2\ )$



Submodular Dictionary Learning

- Monotonic and Submodular Objective Set Function
  - It consists of an entropy rate term $\mathcal{H}(A)$ and a discriminative term $\mathcal{Q}(A)$:

$$\max_{A} \mathcal{F}(A) = \mathcal{H}(A) + \lambda \mathcal{Q}(A) \ s.t. \ A \subseteq E \ and \ N_A \geq K,$$

where

A: selected subset of edge set *E*;

N_A: number of connected components induced by *A*

Submodular Dictionary Learning

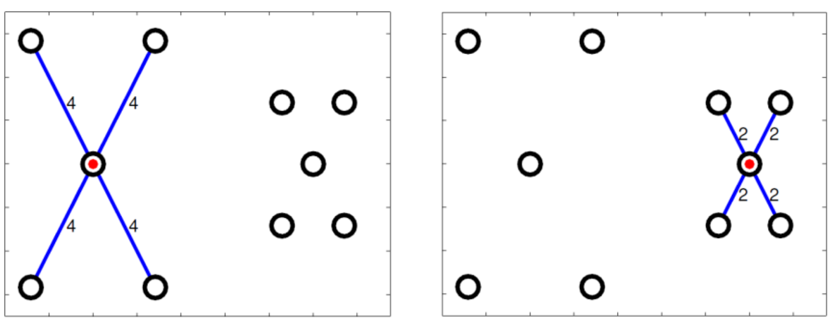
- Entropy Rate of a Random Walk

$$\mathcal{H}(A) = -\sum_i \mu_i \sum_j P_{i,j}(A) \log P_{i,j}(A)$$

μ_i: Stationary probability of vertex v_i

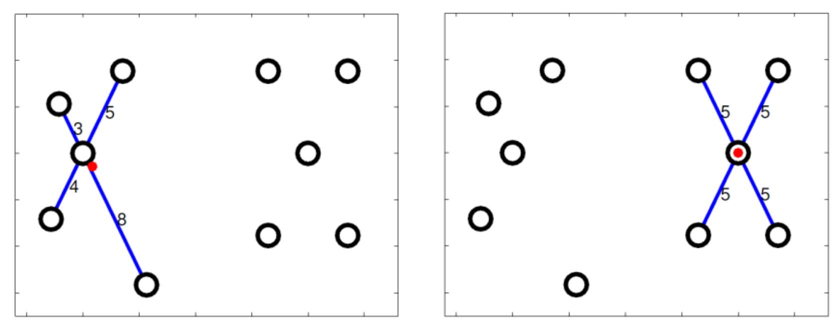
$P_{i,j}$: Transition probability from v_i to v_j

Compactness



(a) Entropy Rate = 0.03 (b) Entropy Rate = 0.43

Homogeneity



(c) Entropy Rate = 0.22 (d) Entropy Rate = 0.24

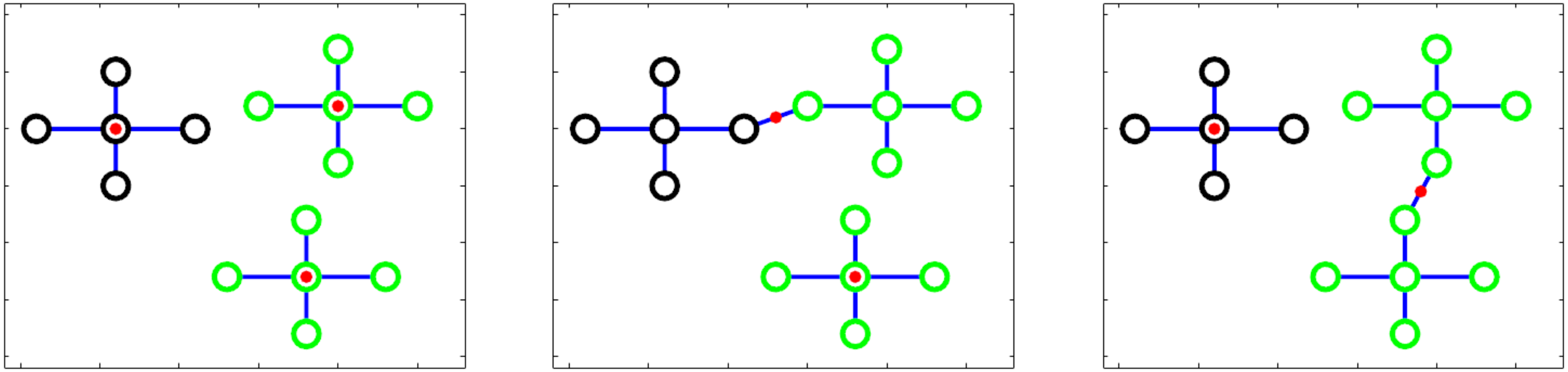
Submodular Dictionary Learning

- Discriminative Term

$$\mathcal{Q}(A) = \frac{1}{C}\sum_{i=1}^{N_A} \max_y N_y^i - N_A$$

N_y^i: Number of elements from class *y* in cluster *i*

Class Pure & A Smaller Number of Clusters



(a) Disc. Fun. = −2.00 (b) Disc. Fun. = −1.33 (c) Disc. Fun. = −1.00

Submodular Dictionary Learning

- Optimization
  - A simple greedy gives a (1/2)-approximation to the optimal solution.

Algorithm 1 Submodular Dictionary Learning (SDL)

Input: $G = (V, E)$, w, K, λ and $\mathcal{N}$
Output: D
Initialization: $A \leftarrow \emptyset$, $D \leftarrow \emptyset$
for $N_A > K$ **do**
  $\tilde{e} = \underset{A \cup \{e\} \in \mathcal{I}}{\operatorname{argmax}} \mathcal{F}(A \cup \{e\}) - \mathcal{F}(A)$
  $A \leftarrow A \cup \{\tilde{e}\}$
end for
for each subgraph S_i in $G = (V, A)$ **do**
  $D \leftarrow D \cup \{\frac{1}{|S_i|} \sum_{j: v_j \in S_i} v_j\}$
end for

Classification

- Object and Face
  - For a test image y_i, first compute its sparse representation:

$$z_i = \arg\min_{z_i} \|y_i - Dz_i\|_2^2 \; s.t. \; \|z_i\|_0 \leq s$$

  - Then the label of y_i is the index i corresponding to the largest element of a class label vector $l = W z_i$.

Multivariate ridge regression

- Human Actions
  - Dynamic time warping is employed to align two sequences in the sparse representation domain; next a K-NN classifier is used

Experimental Results

- Evaluation Datasets
  - Extended YaleB Database (Face database)
  - Keck Gesture Dataset (Gesture)
  - Caltech101 Dataset (Object)
- Experimental Setup
  - Random face-based features

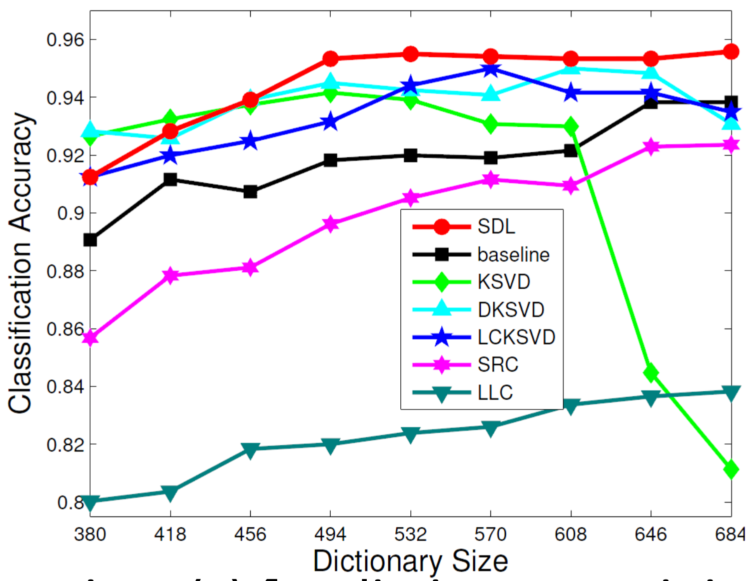
    - dims: 504 (Extended YaleB)
  - Joint Shape and Motion features

    - dims: 512 (Keck Gesture)
  - Spatial pyramid features

    - dims: 3000 (Caltech101)

Experiment Results

- Extended YaleB
  - Classification accuracy comparison

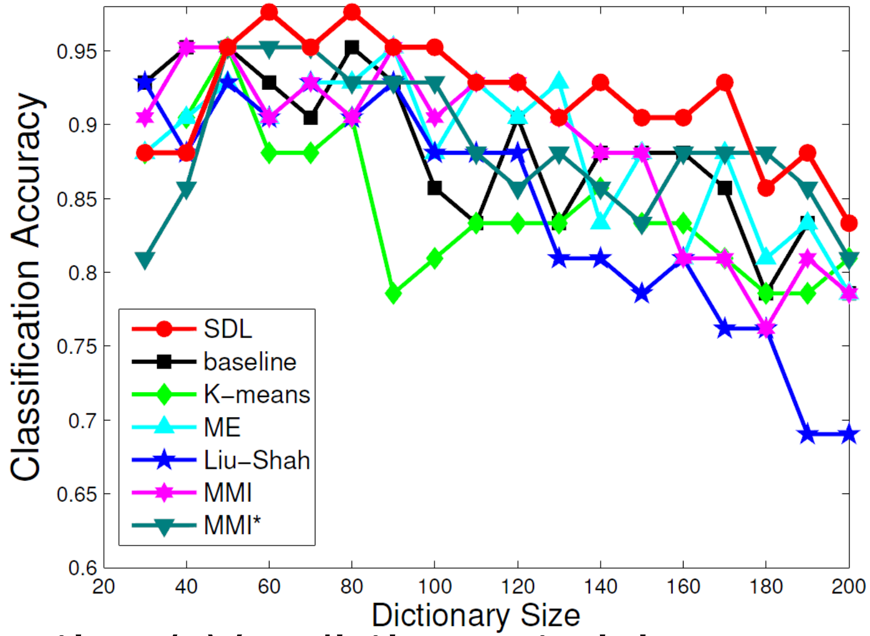


  - Computation time (s) for dictionary training

Dict. size	418	456	494	532	570	608	646	684
SDL	0.9	1.0	0.9	0.9	0.9	1.0	0.9	0.9
K-SVD [1]	52.6	56.1	59.8	64.9	67.9	72.2	76.2	78.0
D-KSVD [35]	53.1	56.9	60.5	65.8	68.1	74.9	77.6	79.2
LC-KSVD [12]	67.2	72.6	78.3	86.5	90.7	97.8	104.4	112.3

Experiment Results

- Keck Gesture Dataset
  - Classification accuracy comparison



  - Computation time (s) for dictionary training

Dict. size	40	60	80	100	120	140	160	180
SDL	1.0	1.0	1.1	1.0	1.0	1.1	1.0	1.0
K-means	1.2	1.1	1.6	1.4	1.8	2.1	2.1	2.2
ME [10]	48.5	57.2	70.2	84.6	91.5	113.1	118.9	130
LiuShah [18]	599.2	597.9	597.2	596.1	593.9	590.3	587.4	582
MMI [26]	64.6	92.6	115.5	140.3	150.1	164.1	184.4	201

Experimental Results

- Caltech101
  - Classification accuracy comparison

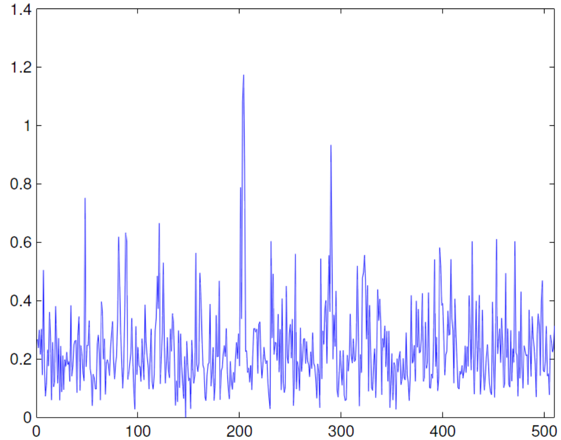
Training Images	5	10	15	20	25	30
Malik [34]	46.6	55.8	59.1	62.0	-	66.20
Lazebnik [15]	-	-	56.4	-	-	64.6
Griffin [9]	44.2	54.5	59.0	63.3	65.8	67.60
Irani [2]	-	-	65.0	-	-	70.40
Grauman [11]	-	-	61.0	-	-	69.10
Venkatesh [25]	-	-	42.0	-	-	-
Gemert [7]	-	-	-	-	-	64.16
Yang [31]	-	-	67.0	-	-	73.20
Wang [29]	51.15	59.77	65.43	67.74	70.16	73.44
SRC [30]	48.8	60.1	64.9	67.7	69.2	70.7
K-SVD [1]	49.8	59.8	65.2	68.7	71.0	73.2
D-KSVD [35]	49.6	59.5	65.1	68.6	71.1	73.0
LC-KSVD [12]	54.0	63.1	**67.7**	70.5	72.3	73.6
SDL	**55.3** ± 0.5	**63.4** $\pm$ 0.5	67.5 $\pm$ 0.3	**70.7** $\pm$ 0.3	**73.1** $\pm$ 0.4	**75.3** $\pm$ 0.4

  - Computation time (s) for dictionary training

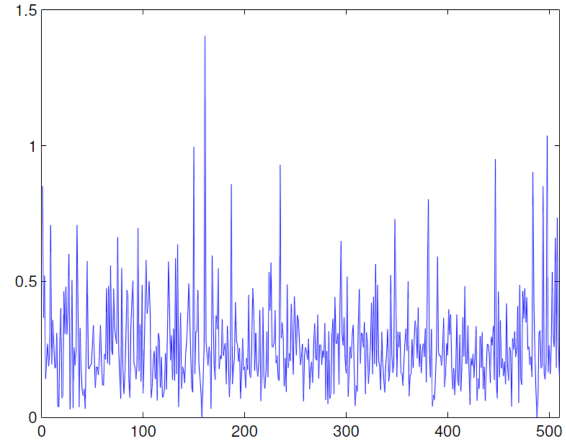
Dict. size	306	510	714	918	1122	1326	1530
SDL	37.5	36.7	36.6	36.9	37.1	36.7	36.7
K-SVD [1]	578.3	790.1	1055	1337	1665	2110	2467
D-KSVD [35]	560.1	801.3	1061	1355	1696	2081	2551
LC-KSVD [12]	612.1	880.6	1182	1543	1971	2496	3112

Experiment Results

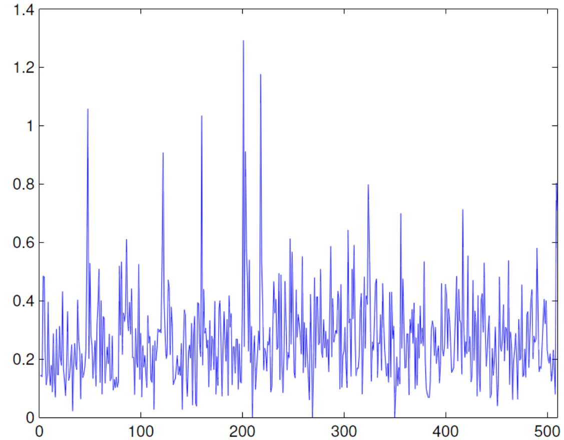
- Examples of sparse codes



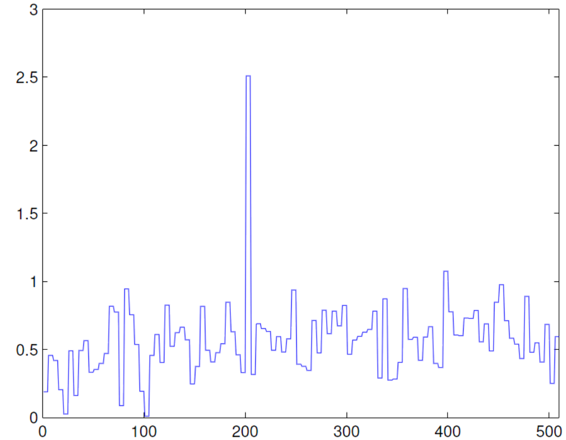
SRC



K-SVD



D-KSVD

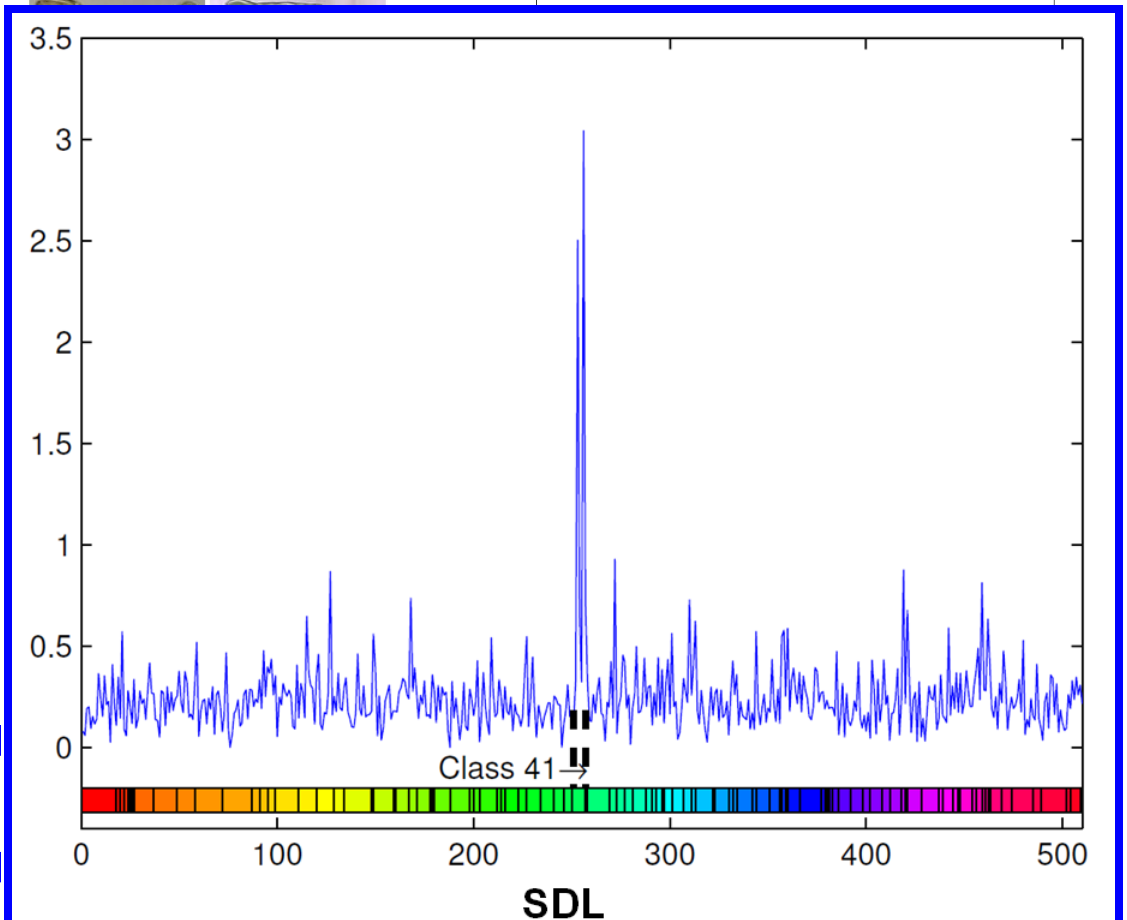


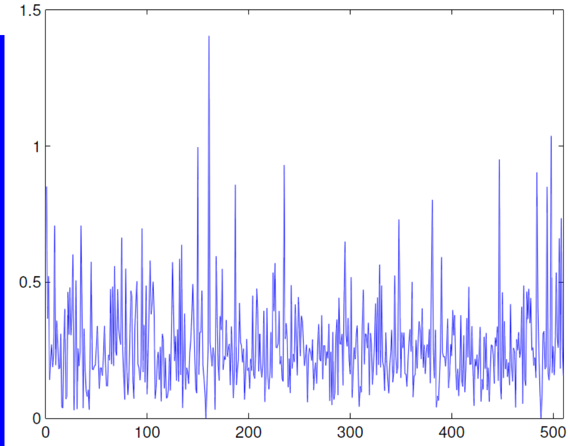
LC-KSVD

- Class 41 in Caltech101 (55 test images).
- Y axis indicates a sum of absolute sparse codes.

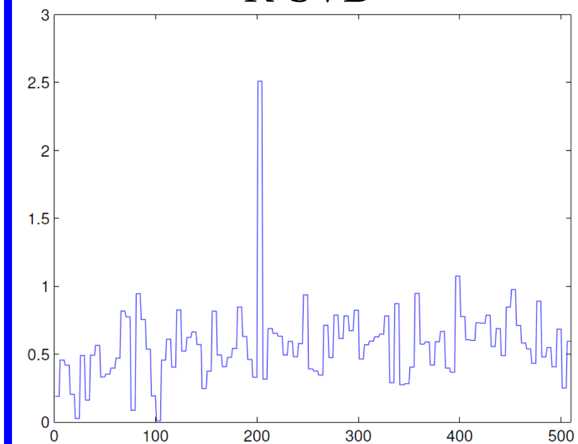
Experiment Results

- Examples of sparse codes





K-SVD



LC-KSVD

Key References

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2. M. Aharon, M. Elad and A. Bruchstein. K-SVD: An algorithm for designing over-complete dictionaries for sparse representation. Sig. Proc., 2006.
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4. M. Liu, O. Tuzel, S. Ramalingam, and R. Chellappa. Entropy rate superpixel segmentation, CVPR 2011.
5. G. Nemhauser, L. Wolsey, and M. Fisher. An analysis of the approximations for maximizing submodular set functions. Mathematical Programming, 1978